

# Constructing more non-singular $Sp$ -branes

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## Abstract

In this note, we construct an array of *non-singular*  $Sp$  (where  $p = D - 4$ ) branes in arbitrary  $D$  dimensions starting from static solutions of black  $p$  brane. These solutions carry nontrivial time dependent profiles of dilaton and associated form fields. We also study other non-singular time dependent configurations. These are obtained from proper analytic continuations of *non-extremal* diholes.

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Non-singular time dependent solutions are very rare in gravity theories, be it supergravity or string theory. Therefore, the recent constructions of non-singular  $Sp$ -brane solutions [1] have attracted much attention [2].  $Sp$  branes are time dependent gravitational configurations with  $p$  longitudinal directions [3]. However, most of these solutions are singular\*. In a recent paper [1], non-singular  $S0$  brane solutions are constructed by applying double-Wick rotations [1] to known black diholes [6]. Analytic continuation of these solutions give exact, time dependent, non-singular  $S0$ -brane in four dimensions. These solutions are non-singular in the sense that, all the conical singularities, which were there in the static solutions of black diholes, can be pushed off to the complex plane of  $\rho$  and  $t^\dagger$ . These solutions often lead to accelerating and bouncing cosmologies. In this note, we generalise these non-singular solutions by analytically continuing  $p$  branes of [7]. We construct  $(D - 4)$  dimensional  $Sp$ -brane solutions in  $D$  dimensions. These are the solutions of  $D$  dimensional maximal supergravity theories and hence can be embedded in string theories. We also construct other non-singular time dependent configurations by analytically continuing the non-extremal diholes of [8], [9].

This letter is organised as follows. First, we construct the non-singular  $Sp$  branes by Wick rotating the solutions of [7]. Here, we give various plots to expose the non-singular nature of the solutions. Subsequently, we analyse the double analytic continuation of *non-extremal* generalisation of black dihole which are different from [1], following the solutions given in [8], [9]. This allow us to construct non-singular  $S0$  branes.

A general class of non-extremal black  $p$  brane solutions in arbitrary  $D$  dimensional space-time was constructed in [7]. The center of the brane sits along the transverse  $z$  axis. We take the linear superposition (unlike the non-linear superposition in the case of non-extremal black dihole) of the  $N$  individual  $p$ -brane potentials to get the potential of  $N$  number of branes along the  $z$  axis. As non-extremal branes exert forces on each other, the superposition of these branes is a tricky issue. This can be achieved however, if many of them are organised on a circle. This has been discussed in [7].

We start with the general supergravity action in  $D$  dimensions of the form

$$S = \int d^D x \sqrt{-g} \left( R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2(p+2)!} e^{-a\phi} F^2_{(p+2)} \right). \quad (1)$$

Here  $\phi$  is the dilaton field, and  $F_{(p+2)}$  is the antisymmetric  $(p+2)$  form field strength. The constant  $a$  denotes the dilaton coupling to the form field  $F_{(p+2)}$ . From this action we can construct a static solution which corresponds to  $N$   $(D - 4)$  branes organised along a common transverse axis. Below, we present the solution; for more detail we refer the reader to the original paper [7].

$$ds_D^2 = e^{\frac{4}{D-2}(U-\tilde{U})} (-e^{2\tilde{U}} d\tau^2 + dx_i dx^i) + e^{2(\frac{D-4}{D-2}\tilde{U} - 4(\frac{D-3}{D-2})U)} [e^{2K} (d\rho^2 + dz^2) + \rho^2 d\theta^2], \quad (2)$$

$$e^{2\tilde{U}} = \prod_{n=1}^N \frac{r_n + \tilde{r}_n - k_n}{r_n + \tilde{r}_n + k_n}, \quad (3)$$

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\*Construction of  $S$ -branes through double Wick rotations are discussed in [4], but all these solutions suffer from singularities. For earlier construction of  $S$  brane solutions, see [5]

<sup>†</sup> $\rho$  is the radial direction of cylindrical polar coordinate system and  $t$  is the time coordinate.

$$e^{2K} = \prod_{n=1}^N \prod_{m=n+1}^N \left[ \frac{r_m \tilde{r}_n + (z - z_m - \frac{1}{2}k_m)(z - z_n + \frac{1}{2}k_n) + \rho^2}{r_m r_n + (z - z_m - \frac{1}{2}k_m)(z - z_n - \frac{1}{2}k_n) + \rho^2} \right] \times \left[ \frac{r_n \tilde{r}_m + (z - z_m + \frac{1}{2}k_m)(z - z_n - \frac{1}{2}k_n) + \rho^2}{\tilde{r}_n \tilde{r}_m + (z - z_m + \frac{1}{2}k_m)(z - z_n + \frac{1}{2}k_n) + \rho^2} \right], \quad (4)$$

$$r_n = \sqrt{\rho^2 + (z - z_n - \frac{1}{2}k_n)^2}, \quad \tilde{r}_n = \sqrt{\rho^2 + (z - z_n + \frac{1}{2}k_n)^2}, \quad (5)$$

$$e^{-\Delta U} = (e^{-\tilde{U}} - c^2 e^{\tilde{U}}) e^{-a^2 \tilde{U}}, \quad \phi_D = 2a_D(U - \tilde{U}), \quad (6)$$

$$A_{0i_1 \dots i_p} = 2ce^{2\tilde{U}}(1 - c^2 e^{2\tilde{U}})^{-1} \epsilon_{i_1 \dots i_p}. \quad (7)$$

In (2)  $\tau, x^i$  are the  $p+1$  world volume coordinates and  $\rho, z, \theta$  are the transverse coordinates. Branes are of masses  $k_n = 2M_n$  (for  $n$ th brane) and centered at points  $z_n = nb$  (that is several branes are arranged periodically along the symmetry axis as discussed earlier).  $r_n$  and  $\tilde{r}_n$  are the distances from the brane center to the field point. Here we would like to make some comments on  $\Delta$ , where  $\Delta = a^2 + \frac{2(p+1)(D-p-3)}{(D-2)}$  according to Ref.[10]. If we consider black hole ( $p=0$ ) solution in pure Einstein-Maxwell theory where there is no dilaton coupling in the action, then  $a=0$  and  $\Delta=1^\dagger$ . We get back the array of standard neutral black hole solution by putting  $c=0$  (in general  $c$  can take values other than zero). Analytic continuation then gives  $S0$ -branes of [1]. For different values of  $a$ , we get different supergravity theories. In this note we will focus on the solutions with single scalar  $\phi_D$ , but there can be other solutions with more than one non-zero scalar field excited.

The double analytic continuation,  $\tau \rightarrow iy$  and  $z \rightarrow it$ , changes the solutions to a time dependent one.

$$ds_D^2 = e^{\frac{4}{D-2}(U-\tilde{U})} (e^{2\tilde{U}} dy^2 + dx_i dx^i) + e^{2(\frac{D-4}{D-2}\tilde{U} - 4(\frac{D-3}{D-2})U)} [e^{2K} (d\rho^2 - dt^2) + \rho^2 d\theta^2], \quad (8)$$

$$e^{2\tilde{U}} = \prod_{n=1}^N \frac{r_n + \tilde{r}_n - k_n}{r_n + \tilde{r}_n + k_n}, \quad (9)$$

$$e^{2K} = \prod_{n=1}^N \prod_{m=n+1}^N \left[ \frac{r_m \tilde{r}_n - (t + i(z_m + \frac{1}{2}k_m))(t + i(z_n - \frac{1}{2}k_n)) + \rho^2}{r_m r_n - (t + i(z_m + \frac{1}{2}k_m))(t + i(z_n + \frac{1}{2}k_n)) + \rho^2} \right] \times \left[ \frac{r_n \tilde{r}_m - (t + i(z_m - \frac{1}{2}k_m))(t + i(z_n + \frac{1}{2}k_n)) + \rho^2}{\tilde{r}_n \tilde{r}_m - (t + i(z_m - \frac{1}{2}k_m))(t + i(z_n - \frac{1}{2}k_n)) + \rho^2} \right], \quad (10)$$

$$r_n = \sqrt{\rho^2 - (t + i(z_n + \frac{1}{2}k_n))^2}, \quad \tilde{r}_n = \sqrt{\rho^2 - (t + i(z_n - \frac{1}{2}k_n))^2}. \quad (11)$$

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<sup>†</sup>for more detail see [11]

This is a non-singular  $Sp$  ( $p = D - 4$ ) brane, as can be seen from the figures below. Here, we plot various metric components for  $N = 2$  and  $\Delta = 4$  (for  $M_n < b$ ). This value of  $\Delta$  appears in the ten dimensional type  $IIA$  supergravity theory where the dilaton coupling  $a$  to the eight form field strength is  $\frac{3}{2}$ ; see for example [12].

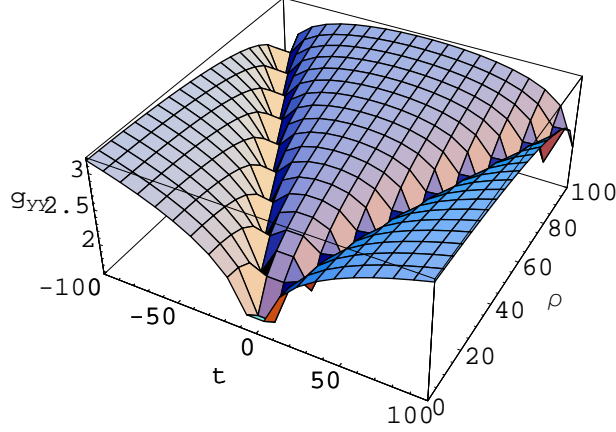


Figure 1: Plot of  $g_{yy}$  vs.  $\rho$  and  $t$  for  $D = 10$ ,  $p = 6$ ,  $b = 1$ ,  $c = 1$  and  $M_n = \frac{1}{2}$

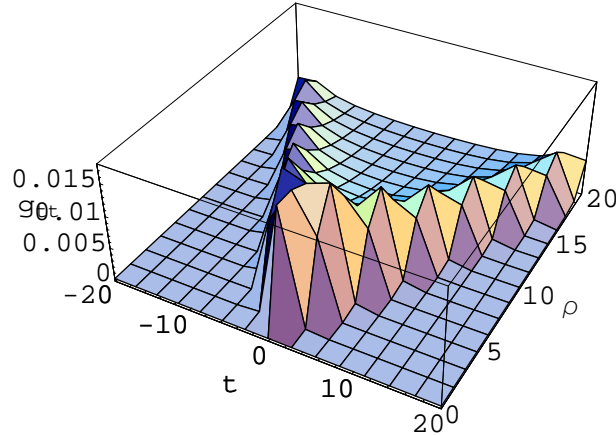


Figure 2: Plot of  $g_{tt}$  vs.  $\rho$  and  $t$  for  $D = 10$ ,  $p = 6$ ,  $b = 1$ ,  $c = 1$  and  $M_n = \frac{1}{2}$

Figure (1), (2), and (3) clearly expose the fact that the various potentials and the metric components in (8)-(10) are smooth for real values of  $\rho$  and  $t$ . Dilaton profile is also non-singular as can be seen from (6). If we take the number of branes  $N \rightarrow \infty$ , then the products in (9) and (10) diverge. To get rid of these divergences from the infinite products we need to multiply divergent constant terms which we call  $e^{-2\tilde{U}_0}$  and  $e^{-2K_0}$  respectively. This would allow us to remove all the conical singularities along the intermediate line between two consecutive branes.  $e^{-2\tilde{U}_0}$  and  $e^{-2K_0}$  can be calculated under the situation where the potential does not depend on the brane transverse direction and the mass of each black branes are equal and are equally spaced along

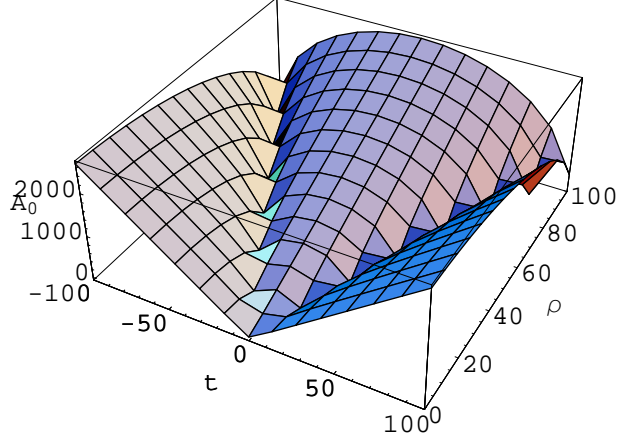


Figure 3: Plot of  $A_0$  (where  $A_0=A_{0,i_1,\dots,i_p}$ ) vs.  $\rho$  and  $t$  for same values of the parameters as mentioned in the previous figure.

their common transverse direction. Here we give the values of  $e^{-2\tilde{U}_0}$  and  $e^{-2K_0}$  following [13] :

$$e^{-2\tilde{U}_0} = \prod_{n=1}^{\infty} \left( \frac{1 + \frac{k}{2nb}}{1 - \frac{k}{2nb}} \right)^2, \quad (12)$$

$$e^{-2K_0} = \prod_{n=0, m=n+1}^{\infty} \left( \frac{(n+m+1)^2 b^2}{(n+m+1)^2 b^2 - k^2} \right)^2. \quad (13)$$

We are therefore able to construct non-singular  $Sp$ -branes by analytically continuing certain static black brane solutions. For particular choices of the dilaton coupling  $a$ , these solutions can be embedded in string theory.

In the next part of this note, we construct some new  $S0$  brane configurations starting from non-extremal black dihole solutions. These non-extremal black dihole solutions are given in [8], [9]. With the choice of the parameters (e.g. mass, charge, angular momentum and shifts along the  $z$  axis of each hole)

$$\begin{aligned} m_1 &= m_2 = m, & q_1 &= -q_2 = q, \\ a_1 &= a_2 = 0, & z_1 &= -z_2 = k, \end{aligned} \quad (14)$$

the axisymmetric solution written in cylindrical coordinate, is

$$ds^2 = -e^{2\tilde{U}} d\tau^2 + e^{-2\tilde{U}} [e^{2K} (d\rho^2 + dz^2) + \rho^2 d\theta^2], \quad (15)$$

$$e^{2\tilde{U}} = \frac{A^2 - B^2 + C^2}{(A + B)^2}, \quad e^{2K} = \frac{A^2 - B^2 + C^2}{gr_+ r_- \tilde{r}_+ \tilde{r}_-}, \quad \Phi = -\frac{C}{A + B}, \quad (16)$$

where  $A, B$  and  $C$  are given as

$$A = -(r_+ - r_-)(\tilde{r}_+ - \tilde{r}_-)[(\kappa_+^2 + \kappa_-^2)(m^4 + \kappa_+^2 \kappa_-^2) - 4m^2 \kappa_+^2 \kappa_-^2]$$

$$\begin{aligned}
& + (r_+ + r_-)(\tilde{r}_+ + \tilde{r}_-)(\kappa_+^2 - \kappa_-^2)(m^4 + \kappa_+^2 \kappa_-^2) \\
& - 2(r_+ r_- + \tilde{r}_+ \tilde{r}_-)(\kappa_+^2 + \kappa_-^2)(m^4 - \kappa_+^2 \kappa_-^2), \\
B & = 4m\kappa_+ \kappa_- (\kappa_+^2 - \kappa_-^2)[(r_+ + r_- + \tilde{r}_+ + \tilde{r}_-)\kappa_+ \kappa_- - (r_+ + r_- - \tilde{r}_+ - \tilde{r}_-)m^2], \\
C & = 4kq\kappa_+ \kappa_- [(r_+ - r_- + \tilde{r}_+ - \tilde{r}_-)(\kappa_+^2 - m^2)\kappa_- - (r_+ - r_- - \tilde{r}_+ + \tilde{r}_-)(\kappa_-^2 - m^2)\kappa_+]. \quad (17)
\end{aligned}$$

The constant  $g$  in the expression of  $e^{2K}$  was determined in [9] by analysing proper location of the conical singularity along  $z$  axis. One can realize this singularity as a strut between the holes and also outside as a string suspended from infinity.  $g$ , computed in [9], is given by

$$g = 64\kappa_+^4 \kappa_-^4 (\kappa_+^2 - \kappa_-^2)^2, \quad (18)$$

$r_\pm, \tilde{r}_\pm$  and  $\kappa_\pm$  are given by

$$r_\pm = \sqrt{\rho^2 + (z \pm \alpha_+)^2}, \quad \tilde{r}_\pm = \sqrt{\rho^2 + (z \pm \alpha_-)^2}, \quad (19)$$

$$\alpha_\pm = \kappa_+ \pm \kappa_-, \quad \kappa_\pm = \frac{1}{2}(\sqrt{m^2 + k^2 + 2k\sqrt{m^2 - q^2}} \pm \sqrt{m^2 + k^2 - 2k\sqrt{m^2 - q^2}}). \quad (20)$$

Black holes are lying along the symmetric  $z$  axis (where  $\rho = 0$ ) centered at the points,  $z = \kappa_+$ . Here,  $2\kappa_+$  is the distance between the center of the two black holes. This distance is controlled by the parameters  $m, k$  and  $q$ .  $\kappa_-$  is the non-extremal parameter in the solutions. In the extremal limit ( $m = q, \kappa_- = 0$ ) the solutions go back to the standard black dihole solutions. In order to obtain the extremal solution, one has to go to a suitable coordinate system [9]. Metric components diverges when  $\kappa_- \rightarrow 0$  in the present coordinate system. Black hole horizons in this case also lie along the  $z$  axis at  $-\kappa_+ - \kappa_- \leq z \leq -\kappa_+ + \kappa_-$  and  $\kappa_+ - \kappa_- \leq z \leq \kappa_+ + \kappa_-$ . In the extremal limit, we get degenerate horizons like black dihole as described in [1].  $r_\pm, \tilde{r}_\pm$  in (19) are the radial distances to the point of observation from the inner and the outer edge of the black holes respectively.

After double Wick rotation ( $z \rightarrow it, \tau \rightarrow iy$ ) the solutions (15)-(19) transformed into the following expressions.

$$ds^2 = e^{2\tilde{U}} dy^2 + e^{-2\tilde{U}} [e^{2K} (d\rho^2 - dt^2) + \rho^2 d\theta^2], \quad (21)$$

$$\begin{aligned}
A & = -(Im\ r)(Re\ \tilde{r})[(\kappa_+^2 + \kappa_-^2)(m^4 + \kappa_+^2 \kappa_-^2) - 4m^2 \kappa_+^2 \kappa_-^2] \\
& + (Re\ r)(Re\ \tilde{r})(\kappa_+^2 - \kappa_-^2)(m^4 + \kappa_+^2 \kappa_-^2) \\
& - 2((Re\ r)^2 + (Im\ r)^2 + (Re\ \tilde{r})^2 + (Im\ \tilde{r})^2)(\kappa_+^2 + \kappa_-^2)(m^4 - \kappa_+^2 \kappa_-^2), \\
B & = 4m\kappa_+ \kappa_- (\kappa_+^2 - \kappa_-^2)[(Re\ r + Re\ \tilde{r})\kappa_+ \kappa_- - (Re\ r - Re\ \tilde{r})m^2], \\
C & = 4kq\kappa_+ \kappa_- [(Im\ r + Im\ \tilde{r})(\kappa_+^2 - m^2)\kappa_- - (Im\ r - Im\ \tilde{r})(\kappa_-^2 - m^2)\kappa_+]. \quad (22)
\end{aligned}$$

Here  $\kappa_+$  and  $\kappa_-$  in (20) certainly do not change under these rotations, but  $r$  and  $\tilde{r}$  change in the following way:

$$r = \sqrt{\rho^2 - (t - i\alpha_+)^2}, \quad \tilde{r} = \sqrt{\rho^2 - (t - i\alpha_-)^2}. \quad (23)$$

The real nature of our solution can be concluded from the following expressions:

$$Re\ r = \sqrt{\frac{1}{2}(\rho^2 - t^2 + \alpha_+^2 + \sqrt{(\rho^2 - t^2 + \alpha_+^2)^2 + 4t^2\alpha_+^2})}, \quad (24)$$

$$Im\ r = \sqrt{\frac{1}{2}(-\rho^2 - t^2 + \alpha_+^2 + \sqrt{(\rho^2 - t^2 + \alpha_+^2)^2 + 4t^2\alpha_+^2})}, \quad (25)$$

$$Re\ \tilde{r} = \sqrt{\frac{1}{2}(\rho^2 - t^2 + \alpha_-^2 + \sqrt{(\rho^2 - t^2 + \alpha_-^2)^2 + 4t^2\alpha_-^2})}, \quad (26)$$

$$Im\ \tilde{r} = \sqrt{\frac{1}{2}(-\rho^2 - t^2 + \alpha_-^2 + \sqrt{(\rho^2 - t^2 + \alpha_-^2)^2 + 4t^2\alpha_-^2})}. \quad (27)$$

Our solutions can be further generalised by considering static non-extremal diholes of four dimensions [9], which have non zero dilaton. Analytic continuation is similar to what we have done here. One then gets non-singular  $S0$  branes with non trivial dilaton profile along with electric/magnetic potentials.

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## References

- [1] G. Jones, A. Maloney, A. Strominger, hep-th/0403050.
- [2] John E. Wang, hep-th/0403094;  
G. Tasinato, I. Zavala, C.P. Burgess, F. Quevedo, JHEP **0404** (2004) 038, hep-th/0403156;  
H. Lu, J.F. Vazquez-Poritz, hep-th/0403248.
- [3] M. Kruczenski, R. C. Myers, A. W. Peet, JHEP **0205** (2002) 039, hep-th/0204144;  
Chiang-Mei Chen, D. V. Gal'tsov, M. Gutperle, Phys.Rev. **D66** (2002) 024043, hep-th/0204071.
- [4] K. Behrndt, S. Foerste, Nucl.Phys. **B430** (1994) 441-459, hep-th/9403179;  
F. Larsen, F. Wilczek, Phys.Rev. **D55** (1997) 4591-4595, hep-th/9610252;  
H. Lu, S. Mukherji, C.N. Pope, Int.J.Mod.Phys. **A14** (1999) 4121-4142, hep-th/9612224.
- [5] E. J. Copeland, A. Lahiri, D. Wands, Phys.Rev.**D50** (1994) 4868, hep-th/9406216;  
H. Lu, S. Mukherji, C.N. Pope, K.-W. Xu, Phys.Rev. **D55** (1997) 7926, hep-th/9610107;  
A. Lukas, B. A. Ovrut, D. Waldram, Phys.Lett. **B393** (1997) 65, hep-th/9608195;  
A. Lukas, B. A. Ovrut, D. Waldram, Nucl.Phys. **B495** (1997) 365, hep-th/9610238;  
E. J. Copeland, J. E. Lidsey, D. Wands, Phys.Rev.**D57** (1998) 625, hep-th/9708154.
- [6] R. Emparan, Phys.Rev. **D61** (2000) 104009, hep-th/9906160.
- [7] H. Lu, C.N. Pope, K.-W. Xu, Nucl.Phys. **B489** (1997) 264-278, hep-th/9609126.
- [8] V. S. Manko, J. Martin and E. Ruiz, J. Math. Phys. **35** (1994) 6644.
- [9] R. Emparan, E. Teo, Nucl.Phys. **B610** (2001) 190-214, hep-th/0104206.
- [10] H. Lu, C.N. Pope, Nucl.Phys. **B465** (1996) 127-156, hep-th/9512012.
- [11] M.J. Duff, H. Lu, C.N. Pope, Phys.Lett. **B382** (1996) 73, hep-th/9604052.
- [12] M. J. Duff, R. R. Khuri, J. X. Lu, Phys.Rept. **259** (1995) 213-326, hep-th/9412184.
- [13] R. C. Myers, Phys. Rev. **D35** (1987) 455.